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TECHNICAL NOTES

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 479

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N O T I C E

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STRENGTH TESTS OF THIN-WALLED DURALUMIN CYLINDERS IN PURE BENDING

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SUMMARY

This report is the third of a series presenting the results of strength tests on thin-walled cylinders and truncated cones of circular and elliptic section; it includes the results obtained from pure bending tests on 58 thin-walled duralumin cylinders of circular section with ends clamped to rigid bulkheads. The tests show that the stress on the extreme fiber at failure as calculated by the ordinary theory of bending is from 30 to 80 percent greater than the compressive stress at failure for thin-walled cylinders in compression. The tests also show that length/radius ratio has no consistent effect upon the bending strength and that the size of the wrinkles that form on the compression half of a cylinder in bending is approximately equal to the size of the wrinkles that form in the complete circumference of a cylinder of the same dimensions in compression.

INTRODUCTION

As part of an investigation of the strength of stressed-skin, or monocoque, structures for aircraft, the National Advisory Committee for Aeronautics in cooperation with the Army Air Corps; the Bureau of Aeronautics, Navy Department; the Bureau of Standards; and the Aeronautics Branch of the Department of Commerce, made an extensive series of tests on thin-walled duralumin cylinders and truncated cones of circular and elliptic section at Langley Field, Va. In these tests, the absolute and relative dimensions of the specimens were varied to study the types of failure and to establish useful quantitative data in the following loading conditions: torsion, compression, bending, and combined loading.

The first and second reports of this series (references 1 and 2) present the results obtained in the torsion and compression tests of cylinders of circular section. The present report is the third of the series, and presents the results obtained in the pure bending tests of cylinders of circular section.

MATERIAL

The duralumin (Al. Co. of Am. 17ST) used in these tests was obtained from the manufacturer in sheet form with nominal thicknesses of 0.011, 0.016, and 0.022 inch. The properties of this material as determined by the Bureau of Standards from specimens selected at random are given in references 1 and 2. Since all the test cylinders failed by elastic buckling of the walls on the compression half of the cylinder at stresses considerably below the yield-point stress, the modulus of elasticity, which was substantially constant for all sheet thicknesses, is the only property of the material that need be considered.

SPECIMENS

The test specimens were right circular cylinders of 7.5- and 15.0-inch radii with lengths ranging from 1.87 to 37.5 inches. The cylinders were constructed in the following manner: First, a duralumin sheet was cut to the dimensions of the developed surface. The sheet was then wrapped about and clamped to end bulkheads. (See figs. 1, 2, and 3.) With the cylinder thus assembled, a butt strap 1 inch wide and of the same thickness as the sheet was fitted, drilled, and bolted in place to close the seam. In the assembly of the specimen, care was taken to avoid having either a looseness of the skin (soft spots) or wrinkles in the walls when finally constructed.

The end bulkheads, to which the loads were applied, were each constructed of two steel plates one-quarter inch thick separated by a plywood core 1-1/2 inches thick for the bulkheads of 7.5-inch radius and 3-1/2 inches thick for the bulkheads of 15.0-inch radius. These parts were bolted together and turned to the specified outside diameter. Steel bands approximately one-quarter inch thick were used to clamp the duralumin sheet to the bulkheads. These bands were bored to the same diameter as the bulkheads.

APPARATUS AND METHOD

The thickness of each sheet was measured to an estimated precision of ±0.0003 inch at a large number of stations by means of a dial gage mounted in a special jig. In general, the variation in thickness throughout a given sheet was not more than 2 percent of the average thickness. The average thicknesses of the sheets were used in all calculations of radius/thickness ratio and stress.

A photograph of the loading apparatus used in the pure bending tests is shown in figure 1. This apparatus is similar to the apparatus used to apply torque to the torsion specimens reported in reference 1. It consists of a rectangular frame with two horizontal beams and two vertical compression members. The lower horizontal beam was attached to the lower bulkhead of the test specimen while the upper horizontal beam was free to rotate about a pin directly above the center of the cylinder. The load was applied by a jack at one corner of the rectangular frame. By means of a flexible cable that passed over a series of pulleys at this corner of the frame, half the load from the jack was transmitted as an up load to one end of the lower horizontal beam while the other half of the load was transmitted through the rectangular frame to the other end of the same beam as a down load. In this manner, a bending moment unaccompanied by transverse shear was applied to the specimen.

In order to determine the possible errors caused by friction in the joints of the frame, a special test was made in which the moment applied to the lower beam was measured directly and compared with the moment calculated from the force applied by the jack. These two moments were found to agree within 1 percent throughout the range of moments applied.

Loads were applied by the jack in increments of about 1 percent of the estimated load at failure. In several instances, preliminary wrinkles began to form on the compression half of the cylinder prior to failure. With increase in load, these wrinkles grew steadily in size and sometimes in number until failure occurred by collapse of the cylinder as characterized by a sudden formation of wrinkles in several circumferential rows. (See fig. 2.) Failure was always accompanied by a loud report and by a reduction in load which continued with deformation of the cylinder after failure. In all the tests, 5 to 10 minutes elapsed from the time that load was first applied to the specimen until failure occurred.

When the cylinder was mounted for test, the seam and butt strap were usually placed at the neutral axis. However, a few tests were made with the seam located on the extreme tension fiber, but the results were in agreement with those obtained when the seam was located at the neutral axis.

DISCUSSION OF RESULTS

In reference 3 Brazier treats the bending of a thin-walled cylinder of infinite length and shows that components of the longitudinal tensions and compressions directed toward the neutral surface when the cylinder is bent cause the cross section to flatten. According to Brazier, the moment resisted by the cylinder passes through a maximum value when the cross section has flattened such an amount that the distance from the neutral axis to the extreme fiber is seven ninths of the radius. Consequently, if the applied moment exceeds the maximum value of the resisting moment, the cylinder of infinite length will collapse by flattening completely at one or more sections. The maximum value of the resisting moment as derived by Brazier is given by the equation

$$M = \frac{2\sqrt{2}}{9} \frac{E \pi r t^2}{\sqrt{1 - \sigma^2}} \tag{1}$$

where M, moment

E, modulus of elasticity

σ, Poisson's ratio

r, mean radius of cylinder

t, thickness of cylinder wall

Unfortunately, there is no theory available concerning the bending of thin-walled cylinders of the lengths tested. For the cylinders considered in this report, failure occurred on the compression half of the cylinder in bending by the formation of wrinkles that were generally similar to those that formed in the complete circumference for cylinders in compression. (See figs. 2 and 3.) Except for the few cylinders in which preliminary wrinkling occurred, there was no visible deformation of the cylinder prior to failure (collapse of the compression half of the cylinder). Consequently, in the analysis of the test results, it is assumed that the ordinary theory of bending applies; i.e., that the stress distribution over the cross section is linear and that the stress on the extreme fiber is given by the equation

$$S = \frac{M\,c}{I} = \frac{M}{\pi\, r^2\, t} \qquad (2)$$

These assumptions may be in error because any tendency of the cylinder to deform between bulkheads prior to failure, even though the deformation be small, will cause the stress distribution to be nonlinear. Consequently, it is proposed to investigate at a later date the stress distribution in cylinders. For the present, however, the results of the tests herein reported are believed to be of sufficient interest to warrant discussion on the basis of the simple bending theory.

Since the type of failure for cylinders in bending is generally similar to the type of failure for cylinders in compression, it is assumed that the compressive stress on the extreme fiber at failure for a thin-walled cylinder in bending is given by an equation of the same general form as the corresponding equation for cylinders in compression (see equation (11), reference 2)

$$S_b = K_b\, E \qquad (3)$$

where S_b is the stress on the extreme fiber at failure as calculated by equation (2) and K_b is a nondimensional coefficient that varies with the dimensions and imperfections of the cylinder.

Values of K_b calculated for each test cylinder are plotted against l/r and r/t in figures 4 and 5, respectively. From these figures it is concluded that, except perhaps for very short cylinders, the radius and thickness as expressed in the ratio r/t are the only dimensions of the cylinder that need be considered in determining K_b, because the large effect of slight imperfections and eccentricities in the elements of the cylinders completely overshadows the small effect of length.

For convenience of reference, the corresponding plot of K_c against r/t for the compression tests made by the N.A.C.A. and reported in reference 2 is given in figure 6. From figures 5 and 6, it is concluded that the stress on the extreme fiber at failure as calculated by the ordinary theory of bending is from 30 to 80 percent greater than the stress at failure for a thin-walled cylinder of the same radius/thickness ratio in compression. This increase in the stress at failure is no doubt caused by an increased stability of the cylinder in bending. The fact that the stress distribution is nonuniform has a tendency to increase the calculated stress on the extreme fiber. In addition, tension in one half of the cylinder provides increased stability for the elements subjected to compression in the other half. The fact that cylinders loaded eccentrically in compression fail at the same calculated stress on the extreme fiber as cylinders loaded centrically (reference 4) indicates that perhaps the presence of tension in one half of the cylinder is the more important of the two factors that contribute toward an increased stability of the cylinder.

The results of the bending tests plotted in figure 5 are replotted in figure 7, where distinction is made regarding those points representing cylinders in which wrinkling occurred prior to failure. From this figure it may be concluded that preliminary wrinkling did not apparently reduce the stress at failure.

After failure the relative shape of the wrinkles is the same for the bending and compression tests. (See figs. 2 and 3.) Consequently, to compare the absolute size of the wrinkles for the bending and compression tests, it is only necessary to compare experimental values of k as defined by the equation

$$k = \frac{2\pi r}{\lambda_c}$$

where λ_c is the wave length of a wrinkle in the direction of the circumference. From table I, where the comparison is made, it is concluded that the size of the wrinkles that form on the compression half of a cylinder in bending is approximately equal to the size of the wrinkles that form in the complete circumference of a cylinder of the same dimensions in compression.

From the preceding discussion it follows that the strength and failure of thin-walled cylinders in bending is closely related to the strength and failure of thin-walled cylinders in compression. For cylinders in compression the results obtained by different investigators were found to differ widely, in some cases, depending upon the technique used in constructing and testing the cylinders (reference 2). Consequently reference 2 should be studied in conjunction with the present report so that proper consideration may be given to the probable dispersion of test results.

In figure 5 are plotted the results of three bending tests reported by Mossman and Robinson in reference 5. In these tests the stress varied between the two bulkheads where failure occurred. Accordingly each experimental point is plotted in figure 5 as a vertical line to show the variation in stress. It will be noted that the results of these test plots are near the lower limit of the results of the N.A.C.A. tests.

There are also plotted in figure 5 the results of 22 bending tests on thin-walled duralumin tubes made by Imperial and Bergstrom. (See fig. 2 of reference 6.) As the stress at failure and the radius/thickness ratio were the only data given in reference 6, it was necessary to divide the former by an assumed value of the modulus of elasticity (10^7 pounds per square inch) to obtain the value of K_b for plotting in figure 5. It will be noted that for values of r/t greater than about 70, the results of the tests plot near the lower limit of the N.A.C.A. results extrapolated to this value. Below $\frac{r}{t} = 70$ the experimental values of K_b tend to be constant, as might be expected, since the stresses that correspond to these values are probably in excess of the yield-point stress for the material.

In reference 6 no stress-strain curves or other information concerning the yield point of the material tested is given. Consequently, it is impossible to draw a general conclusion regarding the bending strength of thin-walled tubes in terms of the properties of the material at the smaller values of r/t. It is significant, however, that the experimental points representing these data indicate an abrupt change from plastic to elastic failure at $\frac{r}{t} = 70$, approximately. This abrupt change is no doubt caused by the fact that the characteristic stress-strain curve for duralumin breaks sharply at the yield point.

CONCLUSIONS

1. The stress on the extreme fiber at failure (collapse of the cylinder) as calculated by the ordinary theory of bending may be given by an equation of the form

$$S_b = K_b E$$

Except for very short cylinders, the radius and thickness as expressed by the ratio r/t are the only dimensions that need be considered to establish the value of K_b.

2. Values of K_b for cylinders in bending are approximately 30 to 80 percent greater than corresponding values of K_c for cylinders in compression.

3. Wrinkling prior to failure did not apparently reduce the stress at failure.

4. After the cylinder has failed the wave lengths of the wrinkles in the direction of the axis and of the circumference are approximately equal and the size of the wrinkles that form on the compression half of the cylinder in bending is approximately equal to the size of wrinkles that form in the complete circumference of a cylinder of the same dimensions in compression.

Langley Memorial Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., September 29, 1933.

REFERENCES

1. Lundquist, Eugene E.: Strength Tests on Thin-Walled Duralumin Cylinders in Torsion. T.N. No. 427, N.A.C.A., 1932.

2. Lundquist, Eugene E.: Strength Tests of Thin-Walled Duralumin Cylinders in Compression. T.R. No. 473, N.A.C.A., 1933.

3. Brazier, L. G.: The Flexure of Thin Cylindrical Shells and Other "Thin" Sections. R. & M. No. 1081, British A.R.C., 1927.

4. Wilson, Wilbur M., and Newmark, Nathan M.: The Strength of Thin Cylindrical Shells as Columns. Bull. No. 255, Eng. Exp. Sta., Univ. Ill., Feb. 28, 1933.

5. Mossman, Ralph W., and Robinson, Russell G.: Bending Tests of Metal Monocoque Fuselage Construction. T.N. No. 357, N.A.C.A., 1930.

6. Younger, John E.: Principle of Similitude as Applied to Research on Thin-Sheet Structures. Aero. Eng., A.S.M.E., Oct.-Dec., 1933, pp. 163-169.

TABLE I

COMPARISON OF EXPERIMENTAL VALUES OF k FOR BENDING AND COMPRESSION TESTS

(Values of k for compression tests obtained from table II of reference 2)

	Radius = 7.5 inches						Radius = 15.0 inches					
l/r	Bending r/t = 323 to 359	Compression r/t = 333 to 362	Bending r/t = 444 to 469	Compression r/t = 455 to 460	Bending r/t = 610 to 688	Compression r/t = 625 to 714	Bending r/t = 673 to 711	Compression r/t = 670 to 757	Bending r/t = 920 to 994	Compression r/t = 909 to 920	Bending r/t = 1364 to 1456	Compression r/t = 1270 to 1415
0.25					16							16
.50	13-14	12	11-13		15-16	13-17		12	14-15		13-14	13-14
.67					11-15	13-15			12-15			12-13
1.00	10-13	9-10	11-12	10	10-13	10-13	8-12	11	11-12	11-13	11-13	12-15
1.50	9-12		10-11		10-11	8-12		10			12-13	
2.00	8-11		9-10		9-11	8-10	8-12		8-13			
2.50						7-10	7-10					
3.00	8-11		8-10		8-11	8-10						
4.00	8-10		7-10									
5.00	6-9											

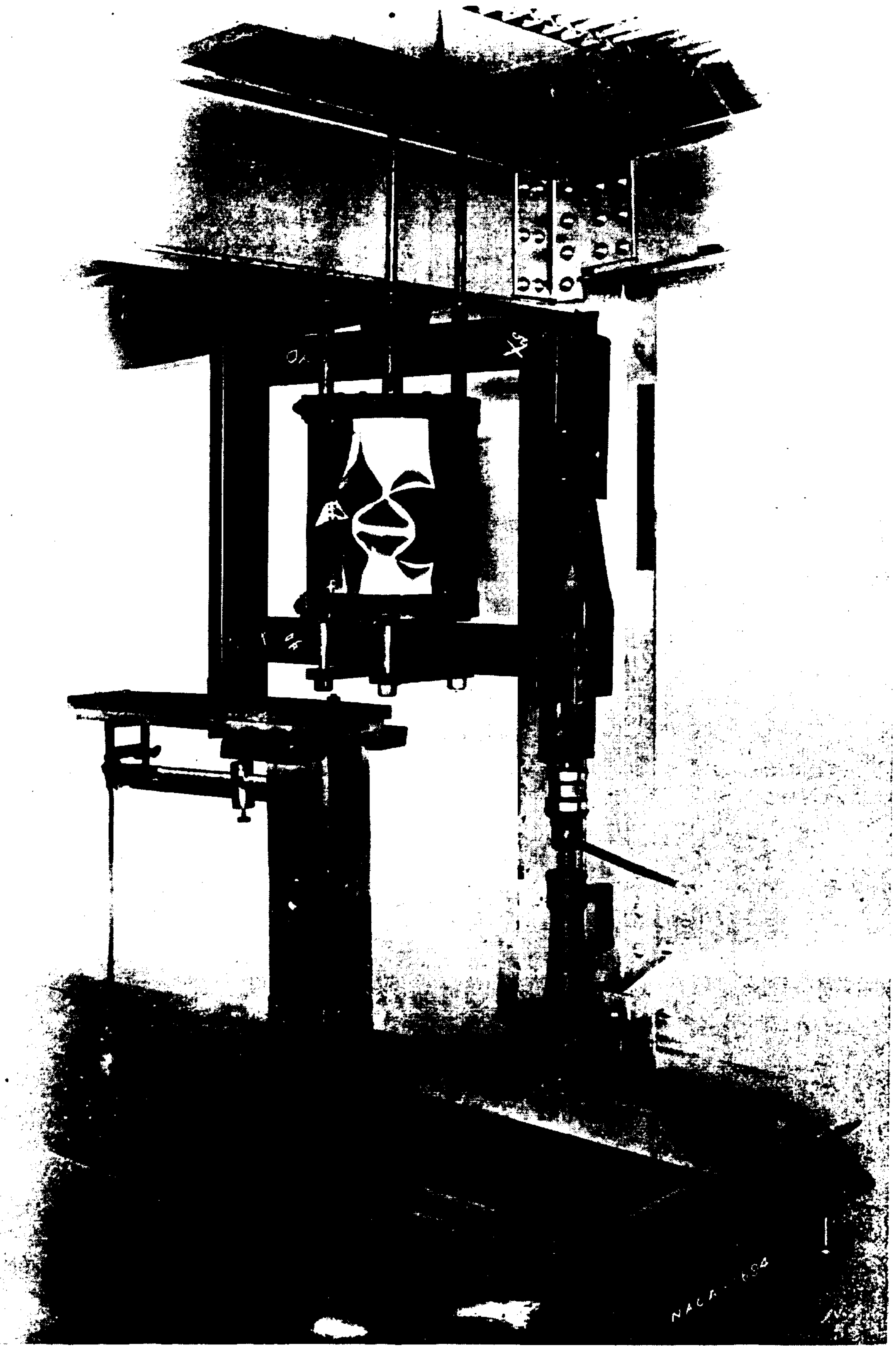
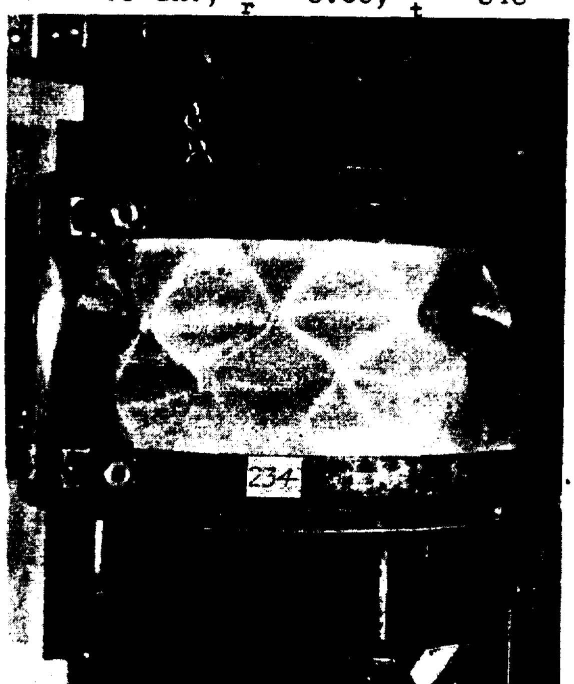


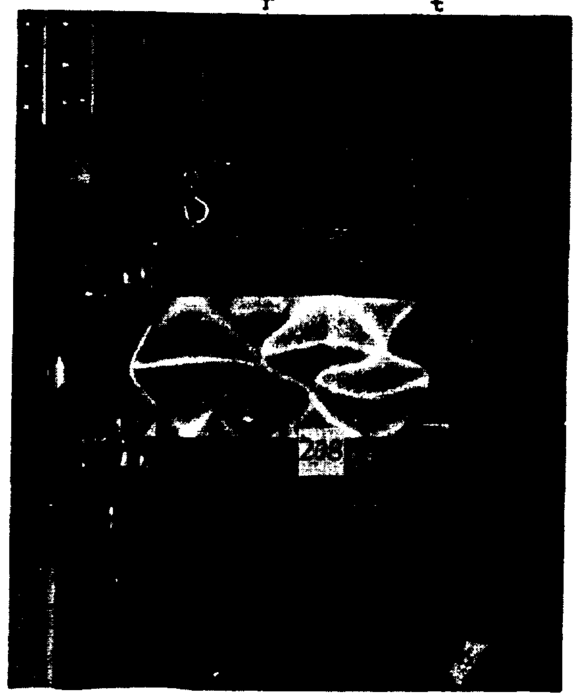
Figure 1.- Loading apparatus used in pure bending tests.

r = 7.5 in.; $\frac{l}{r}$ = 3.00; $\frac{r}{t}$ = 346

r = 15.0 in.; $\frac{l}{r}$ = 2.00; $\frac{r}{t}$ = 673



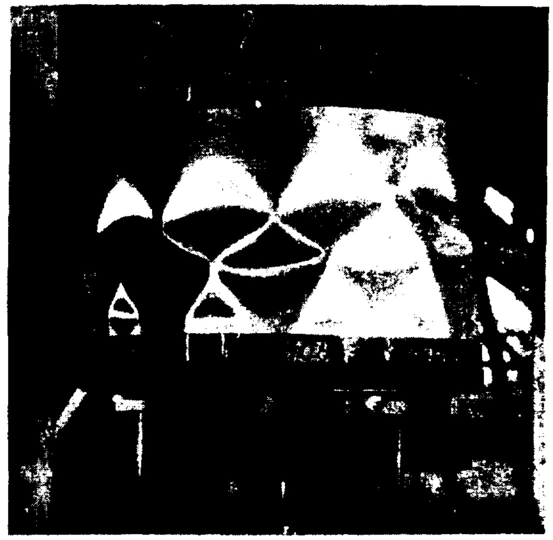
r = 15.0 in.; $\frac{l}{r}$ = 0.70; $\frac{r}{t}$ = 980



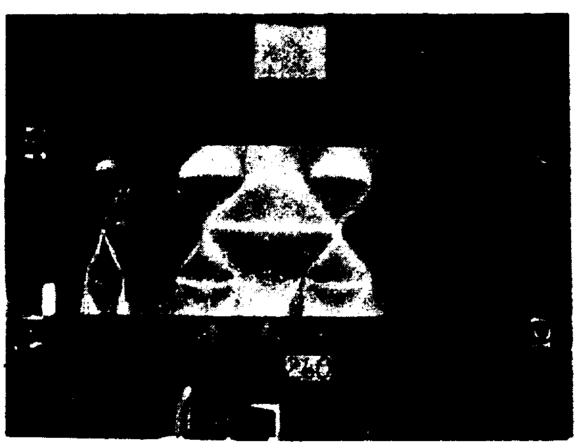
r = 15.0 in.; $\frac{l}{r}$ = 0.50; $\frac{r}{t}$ = 949

Figure 2.- Photographs of cylinders after failure (pure bending tests).

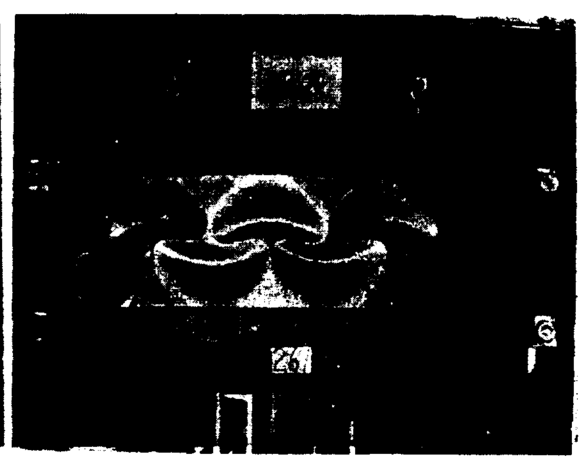
r = 7.5 in.; $\frac{l}{r}$ = 2.50; $\frac{r}{t}$ = 646



r = 15.0 in.; $\frac{l}{r}$ = 1.00; $\frac{r}{t}$ = 920



r = 15.0 in.; $\frac{l}{r}$ = 0.63; $\frac{r}{t}$ = 1,415



r = 15.0 in.; $\frac{l}{r}$ = 0.50; $\frac{r}{t}$ = 711

Figure 3.- Photographs of cylinders after failure (compression tests, fig. 6 of reference 2).

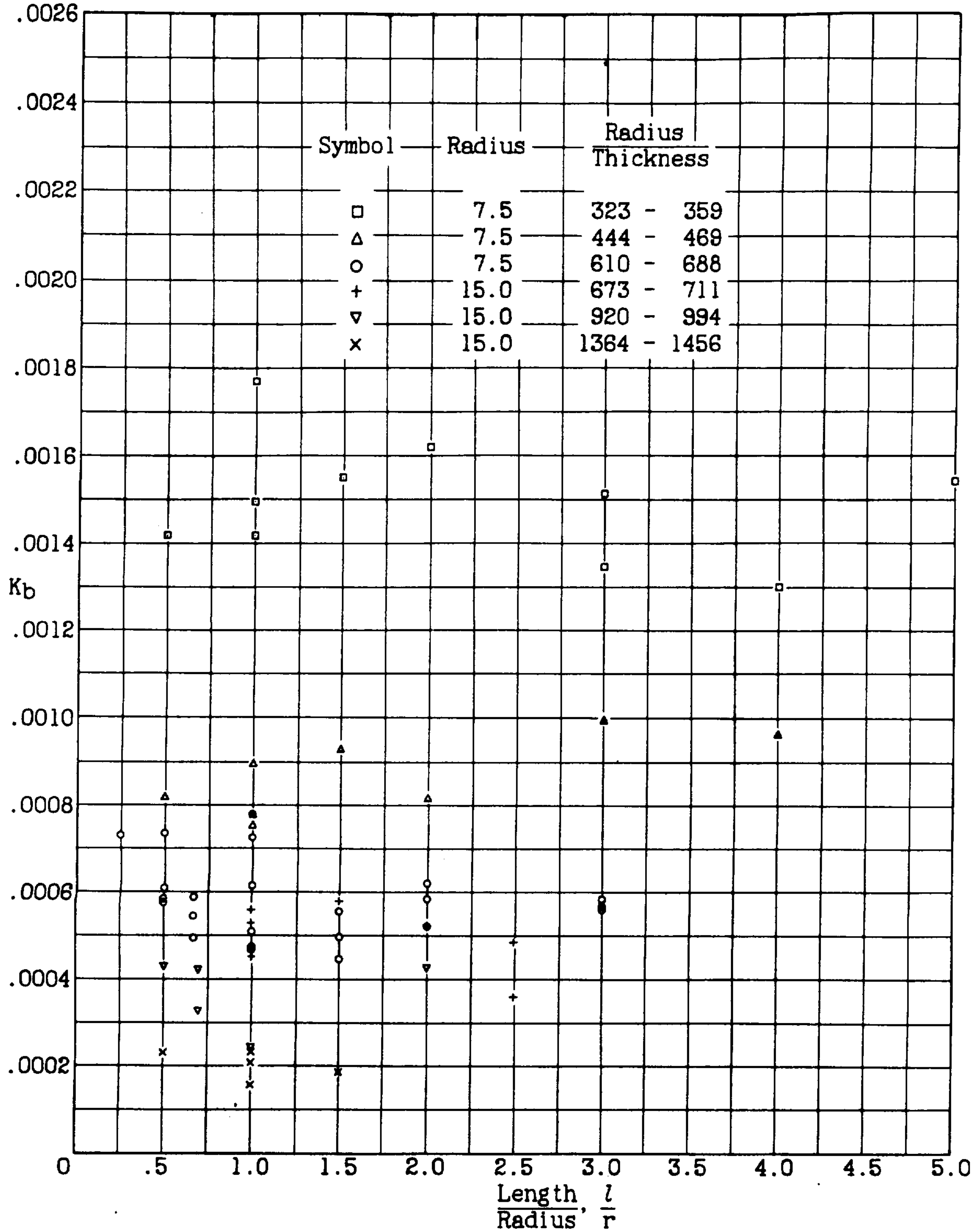


Figure 4.-Plot of K_b against length/radius ratio.

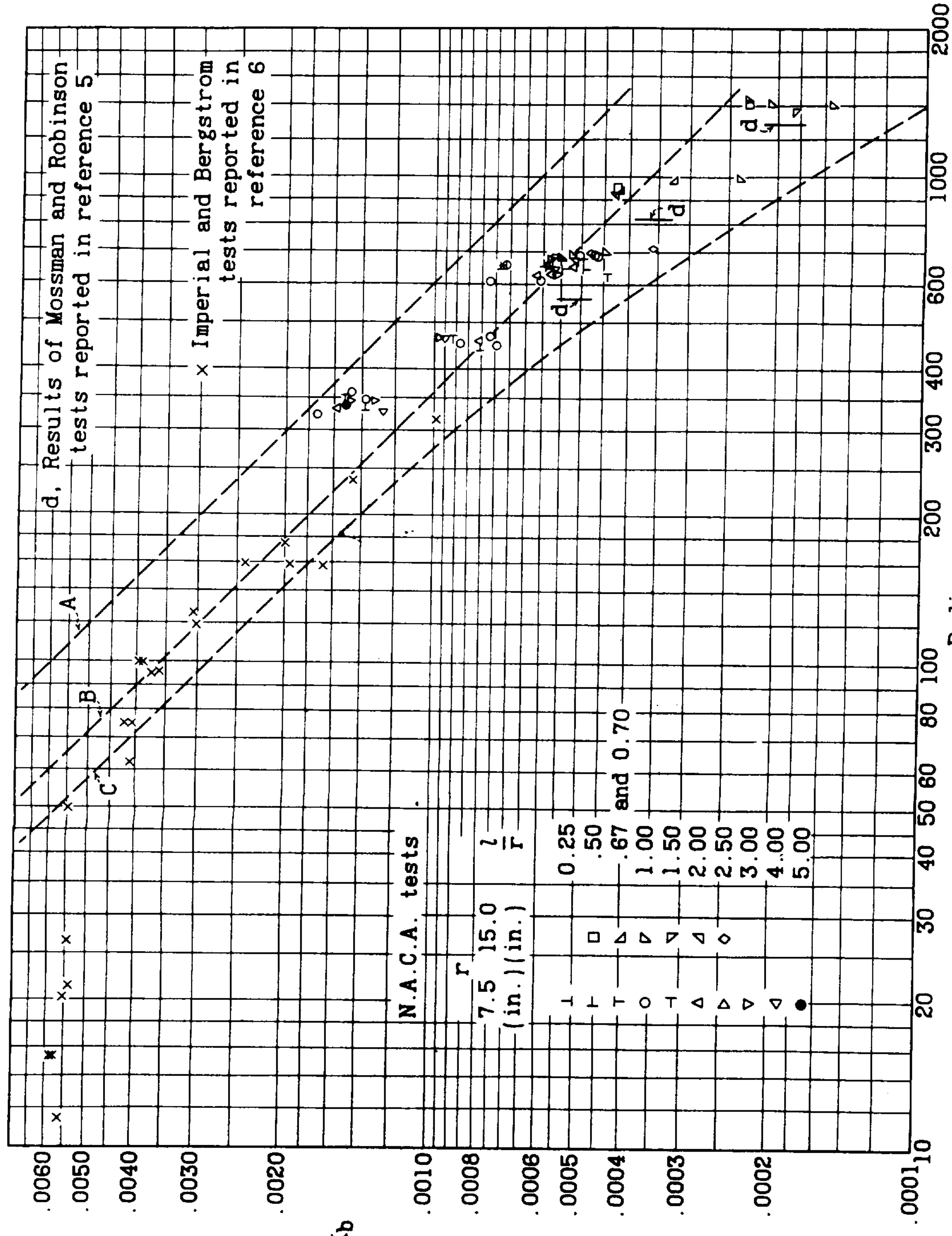
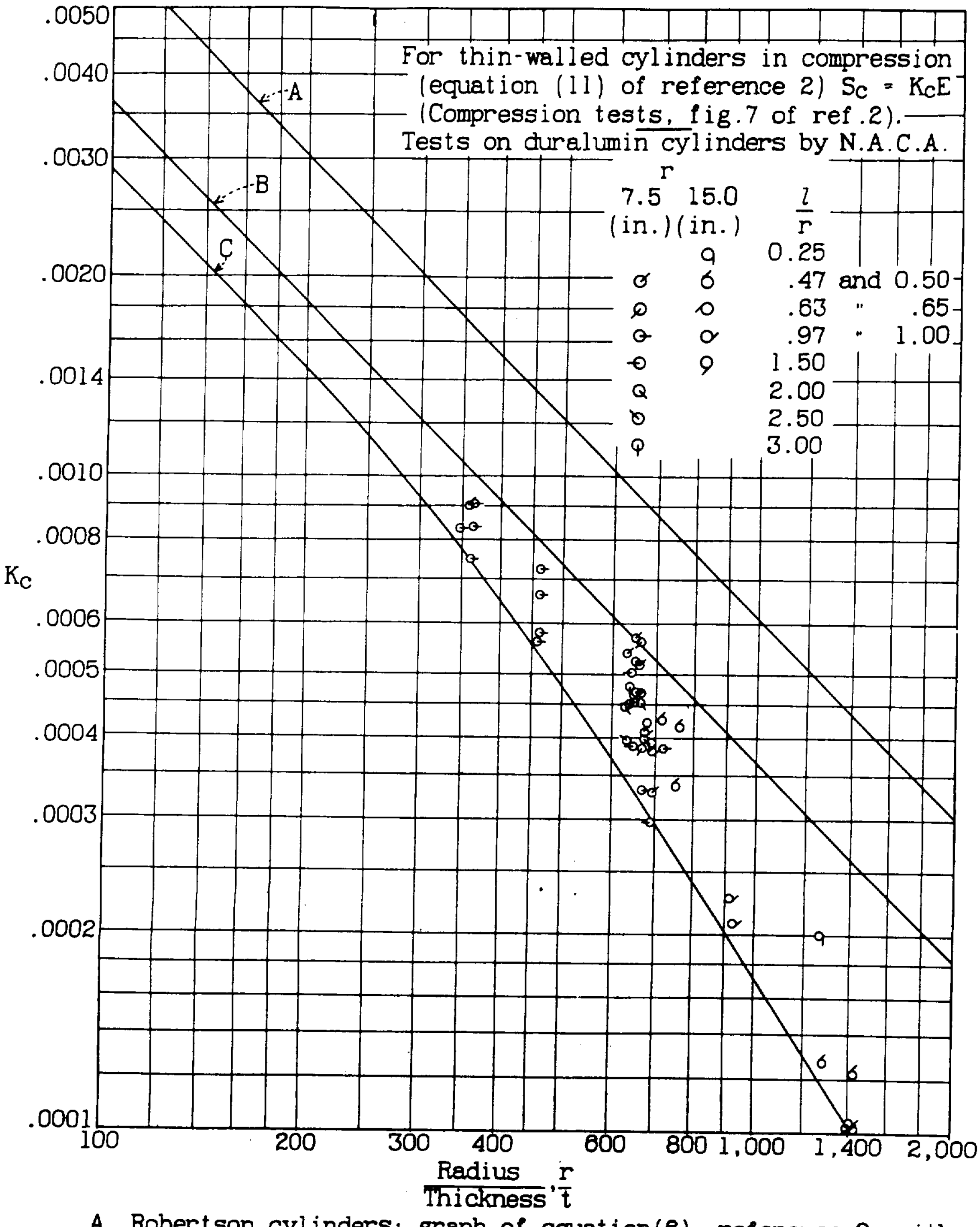


Figure 5.-Logarithmic plot of K_b against radius/thickness ratio (pure bending tests). Curves A,B,and C obtained from figure 6.

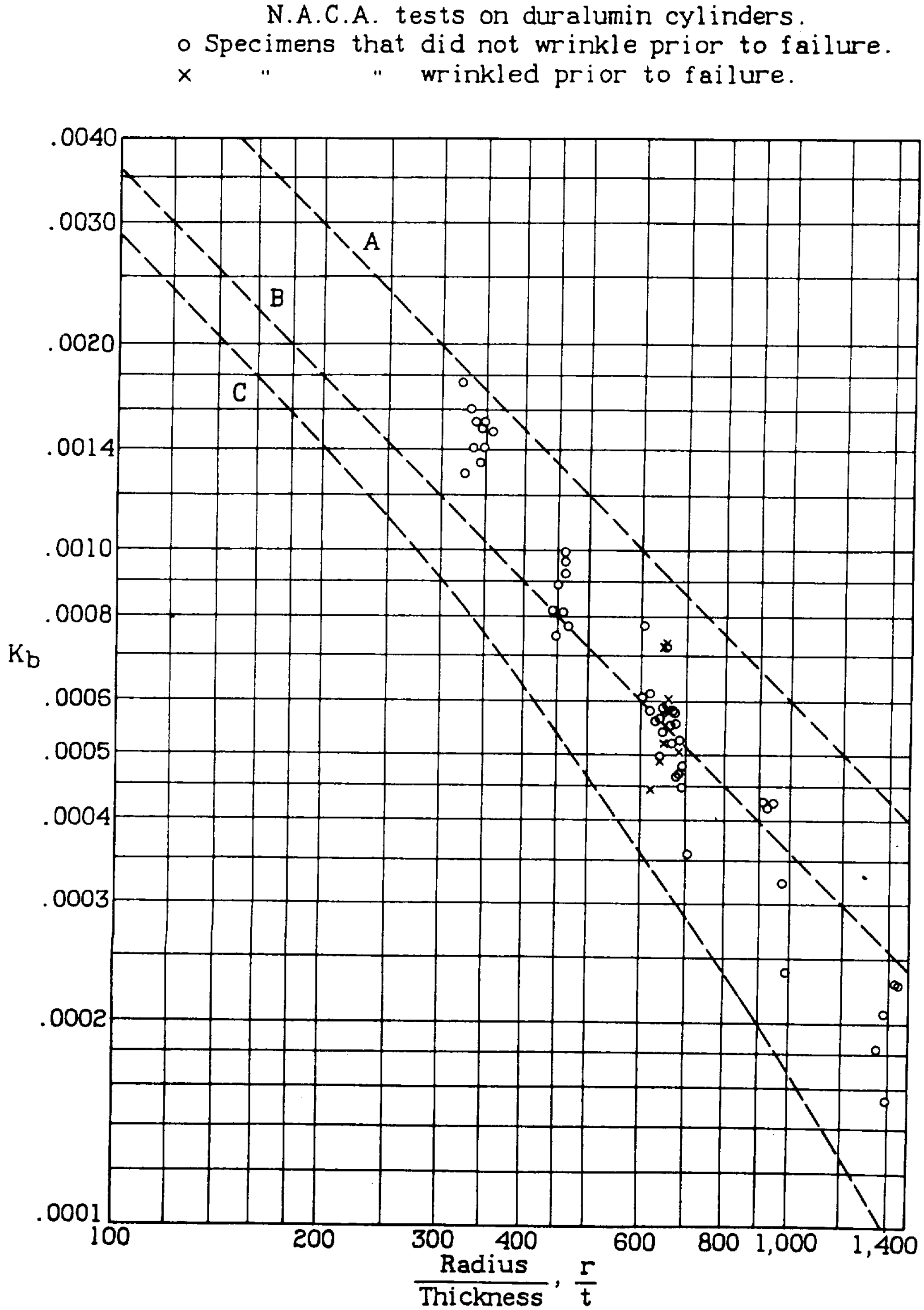


A, Robertson cylinders; graph of equation(6), reference 2, with $\sigma = 0.3$, $\frac{S_{min}}{E} = \sqrt{\frac{1}{3(1-\sigma^2)}}\,\frac{t}{r}$

B, Cylinders of infinite length; graph of equation(8), reference 2, with $\sigma = 0.3$, $\frac{S_{min}}{E} = 0.6\sqrt{\frac{1}{3(1-\sigma^2)}}\,\frac{t}{r}$

C, Lower limit of test data.

Figure 6.-Logarithmic plot of K_c against radius/thickness ratio.



Curves A,B and C obtained from figure 6.

Figure 7.- Effect of wrinkling prior to failure on the bending strength of cylinders.